Three bodies: periodic orbits

READING LEVEL KIDS · EVERYONE · EXPERTS

Computer-assisted certificates for 29 periodic orbits of three equal masses and three at unequal masses, and a conditional KAM result for moth I; checked within this project; not yet refereed.

computer-assisted proof; the program is not published here 32 · Lean-checked 7 · certified 5 · open 5 · certified; the program is not published here 1 · conditional; the program is not published here 1

What we actually checked: that each of the fifteen rows of a 2013 table of repeating three-star orbits, and sixteen more orbits from published lists, is a true repeating orbit inside a tiny stated box, certified (computer-assisted), with seven of the fifteen rows linearly stable and eight unstable.

CHECK IT YOURSELF

The check guide for this book · How to check a result yourself

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1. Three bodies: periodic orbits
2. Articles

Three bodies: periodic orbits

Three stars dancing

The question

Can three stars of the same size dance a pattern that repeats forever? And if they can, how do we know the dance is really there, and not just something a computer rounded into looking right?

My answer: yes. I checked a table of fifteen rows that other scientists found (thirteen different dances), and sixteen more dances. They are all really there. The rest of this page tells you how, and what is still not known.

A puzzle a king paid for

Long ago, in 1885, King Oscar II of Sweden offered a prize: a gold medal and money for anyone who could explain how planets and stars move when they all pull on each other.

Two things pulling on each other are easy. The Earth and the Moon go round and round, the same way every month.

Three things are much harder. Nobody has ever found one simple rule for them. That is called the three-body problem, and people still work on it today.

Pulling, but never touching

Imagine three stars, all the same size. Each one pulls the other two towards it, like invisible elastic bands.

Most of the time the stars get tangled up. They swing close, fling apart, and loop around in a mess that never repeats.

Pulling, but never touching

But sometimes, if they start in just the right places with just the right speeds, they do something beautiful: they dance a pattern, come back to exactly where they began, and dance it again. Forever.

That is called a periodic orbit. Periodic means "it repeats".

Three stars trace a pattern and come back to where they began

Fifteen rows, thirteen dances

In 2013, two scientists, Milovan Šuvakov and Veljko Dmitrašinović, used a computer to search for these dances. They found a whole table of them and gave them names like moth, butterfly, yin-yang and goggles, because of the shapes the stars draw.

Their table has fifteen rows. Some rows are the same dance drawn bigger or smaller, so there are thirteen different dances.

Is the dance really there?

Here is the tricky part. Computers round numbers. Instead of exactly one third, a computer might use 0.3333333. Tiny roundings add up. So a computer can make a dance look perfect when it is really only nearly right.

Is the dance really there?

I used a careful trick. Instead of one number, the computer keeps two: one that is surely too small and one that is surely too big. The true answer is trapped between them, like a ball trapped in a box.

Is the dance really there?

With that trick, I showed that all fifteen rows are really there, so all thirteen dances are, just as the 2013 table said. Each one is trapped in a box so tiny that its width, written as a decimal, starts with twenty-one zeros after the point.

Zooming in on the starting numbers until the tiny box fills the screen

Wobbly or steady?

Next question: if you give the stars a tiny nudge, what happens?

- In a steady dance, the nudge only makes a small wobble. The stars keep dancing.
- In a wobbly dance, the nudge grows and grows until the dance falls apart.

Of the fifteen rows, seven are steady and eight are wobbly.

Two dances side by side: one stays close after a nudge, one drifts away

Question: If we give the moth dance a tiny nudge, what do you think happens?

- It stays close to the first dance.
- It slowly drifts away.

Answer: It stays close. The moth dance is a steady one: a nudge only makes a small wobble. Some other dances are wobbly, and a nudge grows and grows.

[interactive figure: see the web edition]

A tiny difference, a big change

Some motion is so sensitive that a change in the tenth number after the decimal point ends up making a completely different path. Scientists call this chaos. It is one reason the weather is hard to forecast far ahead.

Two butterfly dances that start almost the same, then split apart

The moth that stays

For one dance, moth I, I went further and checked the conditions of a famous result, the KAM theorem (named after Kolmogorov, Arnold and Moser). It says that around a steady dance like this one there are invisible tubes, and stars that start on a tube stay on it forever.

The moth that stays

So moth I is not only steady for a small nudge: there are tubes as close to it as you like, and stars that start on a tube stay on it. That does not mean every nearby motion stays close: between the tubes, motion can slowly wander. This rests on four careful assumptions, written down in the expert version.

Invisible tubes around the moth dance

Sixteen more dances

Other scientists found more dances too. Some are in long lists that Xiaoming Li, Shijun Liao, Ivan Hristov, Radoslava Hristova and their friends published. Their lists pointed the way. Thank you to them.

Sixteen more dances

I checked sixteen of those dances with the two-number trick. All sixteen are really there. Each one is trapped in a tiny box. The box is so tiny that its width, written as a decimal, starts with more than twenty zeros after the point.

Sixteen more dances

Ten came from the published lists. Six more are dances that follow the figure-eight pattern again and again. People had already listed dances that look just like those six, so they are very probably not new; nobody has yet proved that they are exactly the listed ones. I only made sure they are really there.

Sixteen more dances

I also asked: is the dance steady or wobbly? For seven of the ten, the answer is in: three are steady and four are wobbly. For three, the test did not finish. That does not mean steady or wobbly. It means I do not know yet.

I did not check the tubes for these sixteen. That is a job for later. Maybe you will help.

Why checking matters

The king's prize went to Henri Poincaré in 1889. Then Poincaré found a mistake in his own work. He stopped the printing, paid to print it again, and fixed it. His corrected work is where the science of chaos began to grow.

Why checking matters

Great scientists check their own work, and they welcome others checking it too. Šuvakov and Dmitrašinović shared their table so that anyone could build on it. Checking it carefully is a way of saying: this discovery is real, and here is the proof, for everyone.

I give you all the files and a written recipe for each check, so you can check it yourself.

We stand on the shoulders of giants: Poincaré, Šuvakov and Dmitrašinović, Kolmogorov, Arnold and Moser, and many more. Go check yourself.

Where to go next

- What does "steady" or "wobbly" mean for a real dance of three stars? See What was checked.
- What are the invisible tubes around moth I? See Moth I.

Experts: the files and checks are here: check guide.

Yesterday, today, tomorrow

Yesterday, today, tomorrow

Yesterday: In 2013 two scientists, Milovan Šuvakov and Veljko Dmitrašinović, used a computer to find a table of fifteen rows, thirteen different dances that three stars can repeat forever.

Today: I used the careful two-number trick to show that all thirteen dances in the fifteen rows are really there, that of the fifteen rows seven are steady and eight are wobbly, and that moth I is surrounded by invisible tubes, if four careful assumptions hold. I also checked sixteen more dances from other scientists' lists.

Tomorrow: Other people can start from the same tiny boxes and ask the same questions about other dances. They can also check the four assumptions for themselves.

We hope it helps others find more results and keeps their curiosity going. We'd love to collaborate with anyone.

Have an idea or a question? We'd love to hear it! Share it with a parent or guardian, and they can email it to us at hello@gocheckyourself.org.

Articles

- Certified orbits that the catalogues do not list

Each result on its own page

- Proposition 1: the figure-eight, run as a control
- Theorem 2: butterfly I (I.A.1), a periodic orbit with its Morse index and linear stability
- Theorem 3: butterfly II (I.A.2), a periodic orbit with its Morse index and linear stability
- Theorem 4: bumblebee (I.A.3), a periodic orbit with its Morse index and linear stability
- Theorem 5: moth I (I.B.1), a periodic orbit with its Morse index and linear stability
- Theorem 6: moth II (I.B.2), a periodic orbit with its Morse index and linear stability
- Theorem 7: butterfly III (I.B.3), a periodic orbit with its Morse index and linear stability
- Theorem 8: moth III (I.B.4), a periodic orbit with its Morse index and linear stability
- Theorem 9: goggles (I.B.5), a periodic orbit with its Morse index and linear stability
- Theorem 10: butterfly IV (I.B.6), a periodic orbit with its Morse index and linear stability
- Theorem 11: dragonfly (I.B.7), a periodic orbit with its Morse index and linear stability
- Theorem 12: yarn (II.B.1), a periodic orbit with its Morse index and linear stability
- Theorem 13: yin-yang I (II.C.2a), a periodic orbit with its Morse index and linear stability
- Theorem 14: yin-yang I (II.C.2b), a periodic orbit with its Morse index and linear stability
- Theorem 15: yin-yang II (II.C.3a), a periodic orbit with its Morse index and linear stability
- Theorem 16: yin-yang II (II.C.3b), a periodic orbit with its Morse index and linear stability

- Theorem 17: the orbit Hristov list #5, existence and linear stability
- Theorem 18: the orbit Li-Liao I.B_7 (Euler #2023), existence and linear stability
- Theorem 19: the orbit Li-Liao I.A_3 (Euler #4), existence and linear stability
- Theorem 20: the orbit Hristov list #9, existence and linear stability
- Theorem 21: the orbit Li-Liao II.C_2 (Euler #64), existence and linear stability
- Theorem 22: the orbit Hristov list #10, existence and linear stability
- Theorem 23: the orbit Hristov list #29, existence
- Theorem 24: the orbit Li-Liao II.C_61 (Euler #96), existence and linear stability
- Theorem 25: the orbit Hristov list #67, existence
- Theorem 26: the orbit Hristov list #87, existence
- Theorem 27: the orbit that repeats the figure-eight word 17 times, period 116.22, existence
- Theorem 28: the orbit that repeats the figure-eight word 18 times, period 85.49, existence
- Theorem 29: the orbit that repeats the figure-eight word 11 times, period 55.91, existence
- Theorem 30: the orbit that repeats the figure-eight word 10 times, period 77.42, existence
- Theorem 31: the orbit that repeats the figure-eight word 9 times, period 74.73, existence
- Theorem 32: the orbit that repeats the figure-eight word 10 times, period 72.92, existence
- Proposition 33: the thirty orbits are pairwise distinct
- Proposition 34: yin-yang II is linearly stable at both scales
- Proposition 35: a second flow code on the fifteen rows, partial confirmation
- Theorem 36: invariant two-tori accumulate at moth I
- Theorem 37: a KAM criterion applied to the seven hypotheses gives invariant tori
- Theorem 38: the twist determinant alone does not supply the other six hypotheses
- Lemma 39: the endpoint denominators of the twist determinant are positive
- Lemma 40: the endpoints of the twist determinant are in order
- Lemma 41: the upper endpoint of the twist determinant is negative
- Lemma 42: an outward lower bound on the twist determinant
- Lemma 43: an outward upper bound on the twist determinant
- Theorem 44: a periodic orbit at masses 1, 1, 3/2 that the catalogues checked do not list
- Theorem 45: a linearly stable periodic orbit at masses 1, 1, 31/16
- Theorem 46: a linearly stable periodic orbit at masses 1, 1, 97/80, in the family of moth I
- Open question: Morse index and KAM for the sixteen newer orbits
- Open question: Linear stability of three orbits from the published lists

- Open question: Formal identity of the six figure-eight-power orbits with catalogue entries
- Open question: Size of the tori around moth I, and slow drift
- Open question: A search of the rigorous-numerics literature for the sixteen

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