Navier–Stokes: the original 2000 question: what is proved, and what is open

READING LEVEL KIDS · EVERYONE · EXPERTS

Each result is marked proved in writing, Lean-checked (a program that checks every step of a proof), cited, computed or open; the key results were also accepted by two separately written kernel checkers; not yet refereed.

proved in writing 35 · Lean-checked 23 · open 7 · cited 5 · computed; the program is not published here 1

What we actually checked: that under the original 2000 wording of the Navier-Stokes problem a fluid starting at rest in a repeating box has a smooth flow that lasts forever for every allowed push, proved in writing, with every step but one also checked in Lean (that a series converges is checked in writing), and accepted by two other checking programs.

CHECK IT YOURSELF

The check guide for this book · How to check a result yourself

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Navier–Stokes: the original 2000 question:

what is proved, and what is open

The tilted floor

What I found

The 2000 puzzle says the water must move the same way on every tile. It does not say the pressure must. So a swirl that starts still never has to break down, whatever push you give it: the floor can tilt, and the water slides away smoothly, forever. I proved this. Lean checked every step except one, that the long sum in the proof's H^3 bound (a measure of how smooth the water stays) settles to a finite total, which I checked in writing. Two other checking programs, written by different people, agreed with Lean.

Water has rules

When you stir your hot chocolate, or wind blows across a field, the water and air are following rules. Scientists wrote those rules down about 200 years ago. They are called the Navier-Stokes equations, after two scientists: Navier, from France, and Stokes, from Ireland.

The rules work brilliantly. Engineers use them to design planes, boats and weather forecasts.

The million-dollar question

But nobody has shown that the rules always behave.

Could a calm, smooth swirl of water ever twist itself up so tightly that, at one point, its speed becomes infinite? Scientists call that a breakdown.

In 2000 the Clay Mathematics Institute made this one of seven great puzzles. It offered one million dollars for each.

Four doors: A, B, C and D

The puzzle has four doors. Their names are (A), (B), (C) and (D). Open any one door and you have answered the puzzle.

Every door asks about a calm swirl. Two things change from door to door.

Is there a push? Think of pushing a swing.

Four doors: A, B, C and D

- With no push, the door asks: does the swirl always stay smooth, forever?
- With a push, the door asks: can you choose a push that makes the swirl break down?

Where does the water live?

Four doors: A, B, C and D

- In all of space, with no walls.
- In a box that repeats like wallpaper. Picture a floor covered with identical tiles. The water on every tile moves in exactly the same way.

Put the two choices together and you get a square of four doors.

Four doors: A, B, C and D

	No push: does it always stay smooth?	**With a push: can it break?**
All of space	Door A	Door C
Wallpaper box	Door B	Door D

So:

- A and B say: "Calm swirls always stay smooth."
- C and D say: "Some push makes a calm swirl break."

The tilted floor

Water has a speed. It also has a pressure: how hard it is being squeezed and pushed.

The puzzle says the water must be the same on every tile. But as it was printed in 2000, it did not say the pressure must be the same on every tile.

The tilted floor

If the pressure can rise steadily from one side of the floor to the other, it is like tilting the whole floor. Then all the water can simply slide along. The tilt takes up the steady part of a push, and sliding fast smooths out the rest, so the swirl does not have to break down.

Terence Tao noticed this in 2011.

A fix, and a new answer

The puzzle's page also carries a correction (nobody knows exactly when it was added): the pressure must repeat too. No tilting allowed.

In 2026, a team at OpenAI built a push that makes a swirl break down in the wallpaper box, with the pressure repeating too. With the correction, that opens door D. These are OpenAI's results. Grown-up mathematicians still have to check them, and the prize committee has not decided.

Reading the puzzle in different ways

A few sentences in the puzzle can be read more than one way. For example:

- Must the pressure repeat?
- What does "smooth" mean? What does "finite energy" mean?
- Must the swirl be a smooth one, or may it be a rough one?

Reading the puzzle in different ways

I counted every way to mix these readings: more than 440 million. Most ways give the same answer as another way, so the answers fall into a short list.

For each way of reading, each door gets one of three answers:

Reading the puzzle in different ways

- The door opens. The thing the door asks is true.
- The door is locked. The thing the door asks is false, so nobody can prove it.
- Nobody knows.

Finding out that a door is locked is also an answer, because someone has proved it.

Reading the puzzle in different ways

Reading the puzzle exactly as printed in 2000

- Doors A and B: nobody knows. These are the famous hard questions.
- Door C opens with OpenAI's push in all of space. Lean checked this, and another checking program agreed.

Reading the puzzle in different ways

- Door D: a swirl that starts still can slide away from any push, because the floor may tilt. So door D stays shut from rest, for every push. I proved this (the idea is

Terence Tao's), Lean checked it (all but one written step, the H^3 sum), and two other checking programs agreed. For swirls that do not start still, nobody knows.

Reading the puzzle in different ways

Reading it with the correction

- Door D opens with OpenAI's push. Nothing can slide away. Lean checked this too, and two other checking programs agreed.
- Door C still opens with OpenAI's push.
- Doors A and B: nobody knows. I proved that, for door B, the correction does not change the question.

Reading the puzzle in different ways

Reading the correction word for word

Word for word, the correction no longer says the swirl must repeat on every tile. If the pressure need not repeat either, the floor can tilt again, and the swirl slides away: door D stays shut from rest, for any push (I proved this). If the pressure must repeat, a strange swirl might still escape, one that is wilder and wilder far away. Nobody knows yet if one exists. I proved that, for OpenAI's push, it would have to grow very fast far away.

Reading the puzzle in different ways

Other readings

Some readings add a rule the puzzle does not have. They can change the answers a lot:

- Say the swirl must have finite energy even in the wallpaper box. Then door D opens and door B is locked. I proved this, Lean checked it, and another checking program agreed.

Reading the puzzle in different ways

- Say only rough swirls are allowed, called weak ones. Then doors A and B open and doors C and D are locked. I worked this out from famous old theorems by Leray and Hopf. This changes if an extra identity (one more rule about the swirl, spelled out in the book) is also imposed.

Reading the puzzle in different ways

The book lists every reading, with how it was checked: by Lean, by a written proof, or by a famous theorem.

Counting them all

The book gives every way of reading the puzzle its answer and counts them. Experts: the files and checks are here: Every reading.

How it was checked

I used a program called Lean. It checks every single step of a proof, like a very strict teacher who never gets tired. Then two other checking programs, written by different people, checked the most important steps again.

The book lists every result and says which ones Lean checked and which are written proofs.

How it was checked

The Clay Mathematics Institute decides which wording it meant. But anyone can see what is true under each reading, and check it themselves.

We stand on the shoulders of giants: Navier and Stokes, Charles Fefferman who wrote the puzzle, Terence Tao, the OpenAI team, and many more. Go check yourself.

Where to go next

- What is the puzzle about the pressure? See The original 2000 wording: what Tao saw in 2011, and what follows.
- What is still not known? See What remains open.

Yesterday, today, tomorrow

Yesterday, today, tomorrow

Yesterday: Scientists wrote the rules for water and air down long ago. In 2000 Charles Fefferman wrote a very careful puzzle about them, with a big prize, for the Clay Mathematics Institute. Later, Terence Tao noticed that one sentence about pressure could be read two ways. In 2026 a team at OpenAI built pushes that break a swirl: one in all of space, and one in the wallpaper box when the pressure must repeat.

Today: I read the puzzle every way it can be read. A strict checking program, called Lean, helped find out what is true for each way. You will find the answers below.

Tomorrow: Other people can run the same checks themselves.

We hope it helps others find more results and keeps their curiosity going. We'd love to collaborate with anyone.

Have an idea or a question? We'd love to hear it! Share it with a parent or guardian, and they can email it to us at hello@gocheckyourself.org.

Articles

- The Navier–Stokes scorecard: what each wording of Clay's problem answers
- The Pressure Loophole: How the Wording Decides the Answer

Each result on its own page

- Theorem 1: the conditional zero-force drift
- Theorem 2: the bounded drift
- Theorem 3: the slide-through escape
- Theorem 4: the escape-pressure criterion
- Theorem 5: the failure of the escape under repeating pressure
- Theorem 6: the forced breakdown with repeating pressure
- Theorem 7: the two force classes are the same
- Theorem 8: with no force, (B) under the printed pressure and under periodic pressure are the same question
- Theorem 9: the breakdown for every smooth repeating force
- Theorem 10: the breakdown in the Formal Conjectures force class
- Theorem 11: the breakdown for every repeating starting flow
- Theorem 12: the whole-space breakdown
- Theorem 13: Corollary 10.6, the periodic breakdown
- Proposition 14: Tao's Proposition 1.7, elimination of the forcing term
- Lemma 15: the pressure of a smooth periodic solution
- Theorem 16: Tao's Conjecture 1.8 is false as Tao wrote it
- Lemma 17: smoothness on the closed slab
- Lemma 18: Tao's pressure meets each reading
- Lemma 19: the shift lemma
- Lemma 20: the period-defect system
- Lemma 21: the nonzero-defect lemma
- Lemma 22: the subclass-exclusion lemma
- Theorem 23: at most linear growth forces periodicity, the linear-growth repetition theorem
- Theorem 24: for OpenAI's force, growth must be superlinear before time 1, the superlinear-growth theorem
- Proposition 25: the class (iii-a) has fast-growing members
- Proposition 26: uniqueness already fails on a time slab for OpenAI's force
- Proposition 27: the additive shear, a counterexample to the strong form

- Proposition 28: the additive shear, the weaker statement that holds for every periodic U
- Computation 29: the erratum's sign pattern and the defect identities, computed
- Lemma 30: the smooth-solutions identity
- Lemma 31: the extra-identity reduction
- Lemma 32: the curl of OpenAI's force
- Lemma 33: the frozen-vorticity obstruction
- Lemma 34: the shear-start obstruction
- Lemma 35: the witness forces for (D)
- Lemma 36: the whole-space energy exclusion
- Lemma 37: tilt removal while the force is zero
- Lemma 38: the positive-time exclusion
- Lemma 39: the coarser-period reduction
- Lemma 40: the pressure-periodicity lemma
- Lemma 41: the cell-test detection lemma
- Lemma 42: the divergence-free force reduction
- Lemma 43: the equivalence lemma for (B)
- Lemma 44: the pressure-on-all-of-space lemma
- Theorem 45: Seeley's extension theorem
- Theorem 46: global weak solutions
- Proposition 47: the identity algebra of the trace weak class
- Theorem 48: one primitive periodic start settles the weak degenerate-identity groups of (D) and (B)
- Theorem 49: a decaying start and the periodic-pressure obstruction
- Theorem 50: the pressure at time zero only, with smoothness only afterwards, for (A)
- Proposition 51: weak pressure at zero and one-cell identities
- Proposition 52: OpenAI's construction with weak solutions and pressure freedom
- Proposition 53: the minimal-period (D) group, primitive starts
- Theorem 54: the construction's force has exactly zero mean at every time
- Proposition 55: exclusions for OpenAI's construction at the matching viscosity
- Theorem 56: the fixed-viscosity one-cell-identity question is answered yes
- Proposition 57: the three qualifications of the one-cell identity
- Theorem 58: the ordinary printed (C) is answered yes for the checked port, with its variants
- Theorem 59: (C) with weak solutions and the corrected identity tested through time zero
- Theorem 60: OpenAI's construction, weak solutions, zero mean velocity
- Theorem 61: (C) with one force and start for every viscosity: meaning 1 is answered no, by cited small-data theory; meaning 2 is open
- Theorem 62: Clay's (A), (B) and (D) under the 2000 wording, stated directly
- Theorem 63: the additive shear, the strong form for OpenAI's U

- Theorem 64: no additive-shear competitor for OpenAI's force
- Open question: The classical (A) and (B)
- Open question: The literal-erratum question
- Open question: The zero-mean-velocity and cell-energy-inequality questions
- Open question: The sublinear-pressure question
- Open question: The one-cell-identity question with the pressure gradient required to vanish on the cell
- Open question: One force for all viscosities, with the pressure gradient zero
- Open question: Groups of readings decided in part

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