The Invisible Golden Snitch

READING LEVEL KIDS · EVERYONE · EXPERTS

The exact path of every possible Snitch and the corner of their curve: PROVED (written proof, independently re-derived); two chase results: PROVED under stated conditions; sample chases: COMPUTED for speed ratios 2, 3, 5 and 7; catching the Snitch exactly, at every speed ratio above 1: open. Not yet refereed.

proved in writing 4 · cited 1 · computed 1 · proved in writing; the program is not published here 1 · computed; the program is not published here 1 · open 1

CHECK IT YOURSELF

The check guide for this book · How to check a result yourself

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What you can rerun: Every number can be recomputed with your own code from the formulas and inputs in the chapter (from-files). The results-file entries sawtooth_algebra and chase_graded_start are logs-only: the recipe does not give their inputs. Each result page gives its own rerun value and 5-minute check.

What a match shows: a match tells you that the published files really produce the published numbers. It doesn't tell you that the question was the right one to ask, or that a written argument around the numbers is complete. Each recipe says plainly what its steps show and what they don't. For a result whose evidence is logs (logs-only), a match checks the bookkeeping of the logs, not the search that wrote them.

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1 Start here

What this book shows

- My result. While Harry runs straight, each possible Snitch moves along a curve with a closed formula (one you can write down in full): Bouguer's pursuit-curve integral (1732; the classical formula for the path of a chaser), run backwards in time. At time $d/(V - v)$ that curve has a pointed tip at Harry's feet, its two arms leaving at angle $\arccos(1/\alpha)$ (the angle whose cosine is $1/\alpha$) either side of straight behind. PROVED (written proof, independently re-derived).
- Two chase results, under stated conditions. If the uncaught curve is smooth up to Harry, he consumes it at a steady rate; if the Snitches also stay within a half-turn as seen from him, the length left falls by at least $(\alpha - \pi - 1)v$ per unit time. PROVED (written proof, independently re-derived), as an implication. Those conditions are not shown for the real chase, so catching the Snitch for α above $\pi + 1$ is conditional.

- Chases of sample Snitches with the exact motion finish at $\alpha = 2$, 3, 5 and 7 and fail at 1.25 and 1.5. COMPUTED. Catching the Snitch exactly, at every speed ratio above 1, is open.
- Why it matters. Here the target reacts: it steers away from the pursuer, so the places it can be change shape with every step Harry takes. A proof that the smoothness and angle conditions always hold would decide every ratio above $\pi + 1$ at once.
- Every result is listed in the theorem library.

The question

A Golden Snitch hides somewhere on a circle around Harry, and Harry cannot see it. He knows how far away it is and how fast it flies, and he is faster. He does not know which way to go. The moment he moves, the Snitch darts straight away from wherever he now stands. Is there a path that is sure to catch it, whichever way it started?

The riddle comes from Don't Panic Math, in the video "The Invisible Golden Snitch Riddle". Thank you for the question. It sends people to a notebook, and this book is what came out of ours.

[video: The Invisible Golden Snitch Riddle]

The rules are short. Harry flies at speed V and the Snitch at a smaller speed v. Call $\alpha = V/v$ the speed ratio; it is above 1. The Snitch starts at a distance d from Harry, at an angle he is not told. At every instant it flies directly away from the point where Harry is at that instant. Harry sees nothing until he touches it.

"Caught" has two meanings. Catching the Snitch within a small distance ε is disk capture. Catching it exactly is point capture. The video claims disk capture for α above 2π, by a walk along circles that shrink. Point capture for every α above 1 is the question the video leaves open. It is open here too.

Where to go next. What does a search look like when the target cannot be seen? See the question. What if the Snitch ran in a straight line instead? See fog and spirals.

Why it matters

Pursuit is one of the oldest problems in applied mathematics. Pierre Bouguer studied pursuit curves (the paths taken by someone chasing a moving target) in 1732, and for a ship that flees along a straight course in fog, the answer is a logarithmic spiral (a spiral that widens by the same factor each turn; the classical "trawler problem"; see Nahin 2007). Here the target reacts. It steers away from the pursuer, so the places it can be change shape with every step

Harry takes. A search that changes what it searches is harder to picture, and this puzzle is built on that difficulty.

Where to go next. Why does the spiral stop working? See fog and spirals.

What answering it would help with

No use outside mathematics is known to us. Inside it, there are three possible gains.

- A path for every speed ratio above 1 would show that a blind pursuer who is faster always wins against a target that only runs away, and may point to a method for other reacting targets.
- A speed ratio below which no path works would be a threshold, and would mark where blind pursuit stops being possible.
- A proof that the smoothness and angle conditions of chapter 4 always hold would decide every ratio above $\pi + 1$ at once.

Further research is recommended on all three.

Where to go next. Which condition would decide the ratios above $\pi + 1$? See the chase along the edge.

What our research shows

I worked out the exact path of every possible Snitch while Harry runs in a straight line, and I followed those paths to a corner.

The exact path (Sonny, 2026). While Harry runs straight, each possible Snitch moves along a curve with a closed formula. The formula is the classical pursuit-curve integral of Bouguer (1732), run backwards in time. PROVED (written proof, independently re-derived).

The corner (Sonny, 2026). Harry runs straight for the time $d/(V - v)$. At that moment the curve of possible positions has a pointed tip at Harry's feet. Seen from Harry, its two arms leave at angle $\arccos(1/\alpha)$ either side of straight behind, and the distance of the nearest Snitches shrinks like φ to the power $1 - 1/\alpha$. PROVED (written proof, independently re-derived). The

numbers match to better than 2e-7 at a starting angle of 1e-6 radians. COMPUTED.

The tip law and the length balance (Sonny, 2026). If the uncaught curve is smooth up to Harry, he consumes it at a steady rate, and the length left falls by at least $(\alpha - \pi - 1)v$ per unit time whenever the Snitches stay within a half-turn as seen from Harry. PROVED (written proof, independently re-derived), as an implication. The smoothness and the half-turn condition are not shown for the real chase, so catching the Snitch for α above $\pi + 1$ is conditional.

Chases of sample Snitches (Sonny, 2026). With the exact motion and no smoothing, a chase of 1,440 evenly spread sample Snitches finishes at $\alpha = 2$, 3, 5 and 7, in 23.43, 7.21, 2.64 and 1.57 units of d/v. COMPUTED. The times still rise as the sample doubles, so no limit is shown. At $\alpha = 1.25$ and 1.5 every chase we ran fails: the positions run away.

The question itself. Point capture for $1 < \alpha \leq \pi + 1$ is CONJECTURED, with numerical support only from $\alpha = 2$ upward. Point capture at every ratio above 1 is open.

What this is

THE EXACT MOTION OF THE POSSIBLE SNITCH POSITIONS WHILE HARRY RUNS STRAIGHT, THE CORNER OF THEIR CURVE, TWO CONDITIONAL RESULTS ABOUT CHASING ALONG THAT CURVE, AND COMPUTED CHASES OF SAMPLE SNITCHES (SONNY, 2026).

Still open

A PATH THAT CATCHES THE SNITCH AT EVERY SPEED RATIO; THE VIDEO'S CLAIM FOR A ABOVE 2Π; A CATCH OF THE WHOLE CURVE BY ANY OF THE SAMPLE CHASES.

Checked by

DIRECT NUMERICAL INTEGRATION, A FINITE-DIFFERENCE TEST (A NUMERICAL CHECK THAT COMPARES NEARBY VALUES) OF THE STRETCHING IDENTITY, AND CHASES RUN WITH THE EXACT MOTION; AN

INDEPENDENT REPLAY RE-DERIVED THE FORMULAS AND REPRODUCED THE CHASE RESULTS.

Yesterday, today, tomorrow

Yesterday. Bouguer (1732) studied pursuit curves. The classical "trawler problem" gives the spiral answer (see Nahin 2007) for a ship that flees in a straight line. Don't Panic Math put the invisible Snitch to the public, with the rules, a shrinking-circle walk for disk capture and a two-part plan: run out to the circle, then sweep along its edge.

Today. I found the exact path of each possible Snitch while Harry runs straight, found the corner of their curve, and proved two chase results that hold under stated conditions. I ran chases of sample Snitches with the exact motion. They finish from $\alpha = 2$ upward and fail at 1.25 and 1.5.

Tomorrow. Others can try to show the smoothness and angle conditions for the real chase, which would settle every ratio above $\pi + 1$. They can also look for a path that works at ratios near 1, or show that none does. Further research is recommended.

Where to go next. What is proved, and what only under a condition? See what we proved and the chase along the edge.

Check it yourself

What ships: one results file with every number quoted here, a SHA-256 fingerprint (a 64-character code computed from a file; change one bit and the code changes) for it, five figures, and a written recipe for each check. A recipe gives the formula, the numbers to start from and the numbers to expect, so a few lines of your own code will do. The recipes are in the verification guide, which starts with the fingerprint check:

```
sha256sum -c SHA256SUMS.txt  # Linux
shasum -a 256 -c SHA256SUMS.txt   # macOS
CertUtil -hashfile <file> SHA256
```

The upper command runs inside the certificates folder on Linux (macOS: the shasum line). The lower one does the same for one file on Windows. No Lean proof is shipped in this book (Lean is a proof-checking program that tests every step of a proof against fixed rules); the proofs are written and independently re-derived.

The programs are not part of this book.

About this research: https://gocheckyourself.org/about/#how

Where to go next. What does each check show, and what does it leave open? See the verification guide.

The chapters

1. The question
2. Fog and spirals
3. What we proved
4. The chase along the edge
5. The open question

We stand on the shoulders of giants. The credits page lists the work this builds on.

Where to go next

- Coming to the riddle fresh? Start with the question, then fog and spirals.
- Want the results and their limits? See what we proved and the open question.
- Want to rerun a check? See the verification guide.

Have a question about this? Ask it

2 The simple version

The simple version

Ten minutes, no equations. Each part has a short in plain words section and a short for experts note you can skip.

The question

In plain words. Harry is looking for a Golden Snitch he cannot see. It is somewhere on a circle around him, at a distance he knows, and it flies slower than he does. When he moves, it flies straight away from him. Is there a path that is sure to catch it? Don't Panic Math asked this in the video "The Invisible Golden Snitch Riddle". Thank you for the question.

For experts. Planar pursuit with no observations. Harry has speed V and the evader has speed $v < V$, with $\alpha = V/v > 1$. The evader starts on the circle of radius d about Harry at an unknown angle, and moves along the line from Harry to itself, away from him. The question is point capture for every $\alpha > 1$. Disk capture for $\alpha > 2\pi$ by shrinking circles is the video's claim, not proved here.

If the Snitch ran straight out

In plain words. A ship in fog that sails in a straight line is caught by a spiral. The faster ship runs straight out to the circle where the ship would now be, then spirals round on that circle. It meets every ship once. The time is finite for every speed ratio above 1, and huge when the ratio is close to 1.

For experts. The time is $T = \alpha d/(v(\alpha - 1)) \cdot \exp(2\pi/\sqrt{(\alpha^2 - 1)}) - d/v$ (the classical "trawler problem"; see Nahin 2007). This is another model. Here the Snitch flees from Harry's current position, not from his starting point.

What shape the possible places take

In plain words. While Harry flies straight, every possible Snitch follows a path we can write down exactly. Together they form a curve. It starts as a

circle and ends the run as an arrowhead with its point at Harry. The arms of the arrowhead lean back at an angle that depends only on the speed ratio.

For experts. Theorem 2 gives r sin θ = K tan(θ/2)^(1/α), the Bouguer pursuit-curve integral run backwards in time (PROVED: written proof, independently re-derived). At $t_1 = d/(V - v)$ the curve has a corner with cos θ → −1/α, half-angle arccos(1/α) from straight behind, $r_1 \sim C\, d\, \varphi^{(1 - 1/\alpha)}$ and stretch $\sigma \sim (1 - 1/\alpha)\, C\, d\, \varphi^{(-1/\alpha)}$ (PROVED (written proof, independently re-derived), checked to 2e-7 at φ = 1e-6, COMPUTED).

Flying along the edge

In plain words. After the opening run Harry flies along the arrowhead's edge, eating it as he goes. If the edge is smooth and does not turn more than half a turn as seen from Harry, the length left shrinks fast enough to be caught whenever Harry is more than about 4.14 times as fast as the Snitch. Nobody has shown that those two conditions hold during the real chase, so that result is conditional.

For experts. Theorem 5 (tip law) and Theorem 6 (length balance, $dL/dt \le -(\alpha - \pi - 1)v$ under conditions (T) and (A)) are PROVED (written proof, independently re-derived) as implications. (T) is not shown at the corner that starts the chase; (A) is not shown at all.

What the computer says

In plain words. We replaced the curve by a few hundred or a thousand sample Snitches and let Harry catch them one by one, with the exact motion and no smoothing. It finishes when Harry is 2, 3, 5 or 7 times as fast. It does not finish at 1.25 or 1.5: the positions run away. Catching samples is not catching every Snitch, and the times still creep up when we add samples.

For experts. COMPUTED. Times for N = 1,440: 23.43, 7.21, 2.64, 1.57 (α = 2, 3, 5, 7). Smoothed schemes finish at α = 1.5 but move the Snitch positions, so their times are not reported. The one-step sawtooth algebra (Remark 7) is a heuristic and does not separate 1.5 from 2.

What is open

In plain words. Can Harry always catch the Snitch, however little faster he is? That is open. For speeds between 1 and about 4.14 times, we guess yes (CONJECTURED), with computer support only from 2 times upward. For larger ratios the answer is yes if the smoothness and half-turn conditions hold, and nobody has shown them.

For experts. Conjecture 9: point capture for $1 < \alpha \leq \pi + 1$. For $\alpha > \pi + 1$, capture follows from Theorem 6 if (T) and (A) hold throughout.

Experts: the files and checks are here: verification guide.

3 The question

The question

A Snitch you cannot see

Harry stands at the centre of a circle. Somewhere on it is the Golden Snitch, and he cannot tell where. These are the rules, as the riddle sets them.

- Harry flies at a speed V that he knows. He sees nothing until he touches the Snitch, so he plans his whole path in advance.
- The Snitch flies at a speed v, smaller than V. The speed ratio $\alpha = V/v$ is above 1.
- The Snitch starts on the circle of radius d around Harry, at an angle he does not know. He knows d.
- At every instant the Snitch flies straight away from the point where Harry is at that instant. It makes no plan. It only flees.

Harry wins if, at some moment, he and the Snitch are at the same point. The question is whether he can choose a path that wins whichever angle the Snitch started at.

A Snitch dead ahead is the easy case. It moves away at v while Harry closes at V, so the gap shrinks at $V - v$, and he reaches it after $d/(V - v)$. Any

other start is hard, because "away from Harry" keeps turning as Harry moves.

Thank you, Don't Panic Math

The riddle is theirs. Don't Panic Math put it in the video "The Invisible Golden Snitch Riddle". Thank you for a question that rewards a pencil and a long afternoon.

Three things in this book are theirs, as we read the video's slides: the rules above, the walk along shrinking circles, and the two-part plan of running out to the circle and then sweeping along its edge. Every result labelled ours in the later chapters starts from those.

Two ways to catch it

Catching the Snitch within a small distance ε, however small, is disk capture. Catching it exactly is point capture. The two differ in what a miss of one hair's breadth means.

For disk capture the video gives a walk along circles. A lap at radius R takes Harry $2\pi R/V$. In that time the Snitch can move outward by at most $2\pi R/\alpha$, which is less than R exactly when α is above 2π. The video claims that this lets Harry shrink the circle lap after lap until he has swept the Snitch into an ε-disk. We have not proved the claim. It is the video's.

Point capture asks for the exact meeting, for every ratio above 1. The video poses it as open. That is the question of this book, and it is open.

Where to go next. What if the Snitch did not react, and ran straight out? See fog and spirals.

4 Fog and spirals

Fog and spirals

A ship that runs straight out

Picture a ship in thick fog, a known distance d from a faster ship that can see nothing. The far ship steers a straight course away, in a direction nobody knows, at a known speed v. The faster ship wants to catch it. The classical answer is a spiral (the classical "trawler problem"; see Nahin 2007).

The faster ship, call it Harry, runs straight out along one ray. The ship that happened to leave along that ray is caught at once, when Harry reaches the distance where it now sits. That takes the time $d/(V - v)$, and Harry is then at $\alpha d/(\alpha - 1)$ from the start.

Every other ship is now on the circle of that same radius, one for each bearing. Harry turns and keeps his distance from the start equal to the circle's growing radius, $d + vt$. His speed V splits into an outward part v, which keeps him on the circle, and a sideways part $\sqrt{(V^2 - v^2)}$, which carries him round it. After one full turn he has met a ship at every bearing.

Adding the straight run and the turn, the whole chase takes

$$T = \frac{\alpha d}{v(\alpha - 1)}\, e^{2\pi/\sqrt{\alpha^2 - 1}} - \frac{d}{v}.$$

This formula is classical, and it is CITED here (Nahin 2007), not proved in this book. We also checked it against a direct integration of the turn, and the two agree to 15 digits. COMPUTED. With $d = v = 1$, the time is:

Speed ratio α	Time T (units of d/v)
1.25	21,741.37
1.5	826.55
2	74.24
3	12.83
5	3.51
7	1.89

The time explodes as α falls toward 1, because the sideways speed $\sqrt{(V^2 - v^2)}$ shrinks to nothing and the turn takes forever.

[figure: A spiral that catches ships running straight out, at speed ratio 4]

Why the spiral is not the answer

The Snitch of the riddle is not a ship on a straight course. It flies away from where Harry is, and Harry has just moved. A ship on a fixed ray stays on its ray. The Snitch does not, because "away from Harry" points along a different line each time Harry shifts.

So the circle of places the Snitch might be does not stay a circle. As soon as Harry leaves the centre, the circle starts to bend. The next chapter finds the exact shape.

Where to go next. What shape do the possible positions take while Harry runs straight? See what we proved.

5 What we proved

What we proved

One Snitch, exactly

Harry runs along the x-axis at speed V. Measure the Snitch from Harry: r is its distance and θ is its angle from Harry's direction of travel. Harry advances while the Snitch flees along the line from him, so

$$\dot{r} = v - V\cos\theta, \qquad r\dot{\theta} = V\sin\theta.$$

Divide one equation by the other and the unknowns separate. Write u = tan(θ/2). It is a number that starts at 0 when the Snitch is straight ahead and grows without bound as the Snitch swings round behind. Integrating gives

$$r\sin\theta = K\,u^{1/\alpha}, \qquad r(u) = \frac{K}{2}\left(u^{1/\alpha - 1} + u^{1/\alpha + 1}\right),$$

where K depends only on where the Snitch started. The time Harry takes to reach any u is also an exact formula in u. It is Theorem 2 of the theorem list.

This is the classical pursuit-curve integral of Bouguer (1732), run backwards in time. Played in reverse, the Snitch becomes a slower pursuer chasing a target that walks straight, which is Bouguer's setting. We checked the mapping between the two. We did not check it against a printed formula. The statement is PROVED (written proof, independently re-derived). Against direct numerical integration, the closed form differs by at most 1e-13 over 25 start angles and speed ratios. COMPUTED.

The opening run

Harry's plan starts the way the spiral does. He runs straight along one ray for the time

$$t_1 = \frac{d}{V - v}.$$

A Snitch that started straight ahead is caught just as he arrives. Every other Snitch is somewhere on a curve, and the formula above tells us where. Call it the curve of possible positions.

[figure: The circle of possible positions turns into an arrowhead while Harry runs straight, at speed ratio 2]

The corner

At the end of the opening run the curve is no longer round. It has a pointed tip, and the tip is at Harry's feet.

The Snitches that started just beside the ray, at a small angle φ, are the ones Harry nearly met. At the end of the run, each is at an angle θ from Harry's direction of travel, and as φ shrinks, $\cos\theta$ tends to $-1/\alpha$. So the curve leaves Harry along two arms. Each arm makes the angle $\arccos(1/\alpha)$ with straight behind him. For $\alpha = 1.25$ the angle is 36.9 degrees, for $\alpha = 2$ it is 60 degrees, and for $\alpha = 5$ it is 78.5 degrees. For a very large α the two arms flatten into a line.

Two more facts describe the tip. The distance from Harry to the Snitch that started at angle φ shrinks like $\varphi^{(1 - 1/\alpha)}$. The speed at which positions slide along the curve as φ changes grows like $\varphi^{(-1/\alpha)}$, which is unbounded but small enough that the curve still has finite length. Both facts come with a constant $C(\alpha)$ that the theorem list gives exactly.

[figure: The curve at the end of the opening run for three speed ratios, and the distance to Harry near the corner]

The corner is PROVED (written proof, independently re-derived), and the analysis at the tip is ours. At a starting angle of 1e-6 radians, the computed values of $\cos\theta$, of the distance and of the sliding speed match the formulas to better than 2e-7 for all five speed ratios we tried. COMPUTED.

How long the curve is

The curve of possible positions is shorter than the circle it fits inside. We measured it at three speed ratios, in units of d.

Speed ratio α	Length of the curve	Length of the circle
1.25	26.86	31.42
2	11.72	12.57
5	7.68	7.85

These three comparisons are COMPUTED. A proof for every ratio is not in this book.

Where to go next. Once Harry has run the opening run, how does he sweep the curve? See the chase along the edge.

6 The chase along the edge

The chase along the edge

Surfing the edge

After the opening run, the Snitches still at large are the points of the curve that Harry has not yet passed. Picture the curve as a wave front, with Harry at its tip. Every Snitch beyond the tip flees from him, and for a Snitch just beyond the tip, "away from Harry" is along the curve itself. So Harry can walk along the curve, in the direction it points at the tip, and eat it as he goes.

The speed at which he eats it is easy to state. Harry moves at V, the Snitch next to him moves along the same line at v, so the tip advances along the curve at $V - v$. Writing σ for the stretch of the curve (how fast a position slides along it as its label φ changes), the label at the tip moves at the rate $(V - v)/\sigma$.

A second fact keeps this honest. A piece of the curve stretches as Snitches flee, by an amount that depends on its angle to the line from Harry. Close to the tip, that angle is tiny, and the stretching rate falls to zero like one quarter of v times the square of the curvature times the distance from the tip. The tip does not run away from Harry.

Both statements are PROVED (written proof, independently re-derived) under the same tip conditions: the uncaught curve is twice differentiable up to Harry, its curvature at the tip is finite, and the Snitch velocities have a limit at the tip. The real curve starts life with the corner of the last chapter, so these conditions are not shown for the real chase. The exact statement is Theorem 5 of the theorem list.

Counting the length left

Let L be the length of the curve still uncaught. Harry eats $V - v$ of it per unit time. The rest of the curve stretches, by an amount that depends on how

much the curve turns as seen from Harry. Under the tip conditions this gives an exact balance, Theorem 6.

Suppose, as well, that at every moment the directions from Harry to the uncaught Snitches sweep through at most a half-turn and never double back. Then the stretching adds at most πv to the rate, and the length left falls at least at $(\alpha - \pi - 1)v$ per unit time. When α is above $\pi + 1$, the length reaches zero, and Harry has caught every Snitch.

That is PROVED (written proof, independently re-derived) as an implication. The tip conditions and the half-turn condition are not shown for the real chase, so catching the Snitch for α above $\pi + 1$ is conditional on them. Under the tip conditions the same balance gives a floor. The stretching never subtracts, so L cannot fall faster than $V - v$, and a chase of this kind needs at least $t_1 + L/(V - v)$, where t_1 is the length of the opening run and L is the length of the curve at its end.

A warning from the numbers

To see what the real chase does, we replace the continuous curve by sample Snitches: 180, 360, 720 or 1,440 of them, evenly spread over the starting angles. Harry catches them one after another, always flying straight at the next. Every Snitch moves by the exact formula of the last chapter, and nothing is smoothed.

[figure: Harry chasing 180 sample Snitches along the edge of the curve, at speed ratio 3]

At speed ratios 2, 3, 5 and 7 the chase finishes. Times are in units of d/v and include the opening run.

Speed ratio α	180 samples	360	720	1,440	Largest total turn of the directions to the Snitches, at 1,440
2	22.53	22.91	23.20	23.43	2.92
3	7.07	7.13	7.18	7.21	3.02
5	2.614	2.628	2.636	2.641	3.07
7	1.555	1.561	1.565	1.567	3.08

This is COMPUTED for these sample sets. Two things stand out. The times still rise each time the sample doubles, so no limit is shown. And the largest total turn stays below π, which is 3.14, in every finished run, but creeps toward it as the sample grows. Catching 1,440 sample Snitches is not catching all of them. A Snitch between two samples can slip through.

At speed ratios 1.25 and 1.5, every chase we ran fails. The positions run away to huge distances, at every sample size we tried, from 180 to 800, and on a second layout of samples that crowds them toward the corner. We report no capture time at those ratios.

Why? Here is a heuristic, and only that. Write the update of a small wobble near the tip, in a simple forward step, as a one-step rule. The tip's own coefficient is $-\alpha/(\alpha-1)$, and a wobble that alternates sign from one sample to the next is multiplied by $-(4\alpha-1)/(2(\alpha-1))$. Both are larger than 1 in size at every α, so neither one explains by itself why 2 works and 1.5 does not.

Two other schemes do finish at 1.5. One that averages each point with its neighbours at every step gave a time near 100, and a coarser integrator gave a time near 180. Averaging moves the Snitches, so those numbers are not capture times for any real path. The exact motion gives neither.

Where to go next. What would it take to turn these conditions and counts into a complete answer? See the open question.

7 The open question

The open question

What is open

Can Harry catch the Snitch exactly, whatever the starting angle, at every speed ratio above 1? This is open.

The pieces around it are in different states.

- For ratios from 1 up to $\pi + 1$ (about 4.14), the answer yes is CONJECTURED. The numerical support starts at $\alpha = 2$ and goes upward: sample chases finish at 2 and 3. Below 2 there is none, and every unsmoothed chase we ran at 1.25 and 1.5 fails.
- For ratios above $\pi + 1$, Theorem 6 gives capture if the tip conditions and the half-turn condition hold at every moment of the chase. The conditions are not shown, so the claim is conditional and the question is open.
- For disk capture at ratios above 2π, the walk along shrinking circles is the video's claim. We have not proved it.
- For ships that run straight out, a different model, the spiral catches every ship at every ratio above 1. That is classical.

[figure: What is known about catching the Snitch, by speed ratio]

Where to go next

Five places to push, from the nearest.

1. The half-turn condition. Show that, along the real chase, the directions from Harry to the uncaught Snitches never turn by more than half a turn in total. Together with the tip conditions, this would settle every ratio above $\pi + 1$. A counterexample would show where the length argument breaks.
2. The corner at the start. The tip conditions fail at the very beginning, where the curve has the corner of chapter 3. Show that the curve is smooth enough a moment later, or find what replaces the tip law at a corner.
3. Ratios near 1. Find a path that works at 1.5, or show that the failures at 1.25 and 1.5 are real and not an artefact of how we step. A scheme that moves the Snitches, such as averaging, cannot answer this.
4. The whole curve. Turn the sample chases into a statement about every Snitch, for example by a bound on the Snitches between two samples.

1. The video's claim. Prove, or refute, that the walk along shrinking circles gives disk capture for every ratio above 2π.

Every number in this book can be rechecked from the verification guide, and every statement has its exact wording in the theorem list.

Thank you again to Don't Panic Math, for a riddle worth the notebook: "The Invisible Golden Snitch Riddle".

8 Statements

Statements

Each statement starts with the question it answers, then the answer, then how it was checked and on what conditions. "Checked" means by the recipes in the verification guide, on the numbers in the results file in the certificates folder, unless a line says otherwise.

Can Harry catch the Snitch exactly, at every speed ratio above 1?

Answer. Not shown here (status OPEN). The riddle is posed by Don't Panic Math in the video "The Invisible Golden Snitch Riddle". The rules: Harry flies at speed V and sees nothing; the Snitch starts at distance d at an unknown angle and flies at speed $v < V$ directly away from Harry's current position; $\alpha = V/v > 1$.

What is the exact path of a possible Snitch while Harry runs straight?

Answer. With $u = \tan(\theta/2)$ and r, θ measured from Harry, the path satisfies $r\sin\theta = y_0 u_0^{-1/\alpha} u^{1/\alpha}$, and the time to reach u is an exact algebraic expression in u (Theorem 2). Status PROVED (written proof, independently re-derived). The integral is Bouguer's (1732), run backwards in time.

Range. Harry on a straight line at constant speed, any $\alpha > 1$, a Snitch that is not yet caught. Checked against direct numerical integration in 25 cases; the largest difference is 9.9×10^{-14} (COMPUTED).

What does the curve of possible positions look like at the end of the opening run?

Answer. After the time $t_1 = d/(V - v)$ the curve has a corner at Harry's position. Its two arms leave at the angle $\arccos(1/\alpha)$ from straight behind Harry. The distance from Harry of the Snitch that started at angle φ is $C(\alpha)\, d\, \varphi^{1 - 1/\alpha}(1 + o(1))$ as $\varphi \to 0^+$ (Theorem 3). Status PROVED (written proof, independently re-derived).

Range. Every $\alpha > 1$. Checked at $\varphi = 10^{-6}$ for five ratios to better than 2×10^{-7} (COMPUTED). The curve is shorter than the circle it fits in at three ratios (Result 4, COMPUTED); a proof for every ratio is not in this book.

Can Harry consume the curve from its tip?

Answer. Under conditions (T) of Theorem 5, the label of the tip moves at the rate $(V - v)/\sigma$, and the stretching rate at the tip tends to 0. Under (T) and the half-turn condition (A) of Theorem 6, the length left falls at least at $(\alpha - \pi - 1)v$ per unit time, so the Snitch is caught if $\alpha > \pi + 1$. Status PROVED (written proof, independently re-derived) as an implication.

Range. The conditions (T) and (A) are not shown for the real chase. Catching the Snitch for $\alpha > \pi + 1$ is conditional on them.

Does a chase of sample Snitches finish?

Answer. With the exact motion and no smoothing, a chase of N evenly spread sample Snitches finishes at $\alpha = 2, 3, 5, 7$ for $N = 180, 360, 720, 1440$ (Result 8). It does not finish at $\alpha = 1.25$ or 1.5 at any sample size tried. Status COMPUTED.

Range. These sample sets. The times rise as N doubles, so no limit is shown, and catching samples is not catching every Snitch.

Can Harry catch the Snitch exactly for $1 < \alpha \leq \pi + 1$?

Answer. Yes, as a conjecture (Conjecture 9). Status CONJECTURED. The numerical support starts at $\alpha = 2$. There is none below $\alpha = 2$.

9 Theorems in this book

Theorems in this book

Every theorem, remark and computed result in this book, with its exact statement, status and credit. Maths is written in LaTeX notation, between dollar signs.

Setup used by every entry. Harry $H(t)$ moves at constant speed V. A Snitch $S(t)$ moves at constant speed $v < V$ directly away from Harry: $\dot{S} = v\,(S - H)/\parallel S - H \parallel$. The speed ratio is $\alpha = V/v > 1$. At time 0 Harry is at the origin and the Snitch is at $d(\cos\varphi, \sin\varphi)$ for an angle $\varphi \in [0, 2\pi)$ that Harry is not told. Catching means $S(t) = H(t)$ for some t. Statuses are PROVED (written proof, independently re-derived), COMPUTED, CONJECTURED and CITED. Where a result holds only under conditions, the conditions are written into its statement.

Proposition 1: the spiral against ships that run straight out (the classical "trawler problem"; see Nahin 2007)

Credit: the classical "trawler problem" (see Nahin 2007), for the logarithmic-spiral answer. Cited, not proved in this book.

In a different model, each ship $s_\varphi(t) = (d + vt)(\cos\varphi, \sin\varphi)$ runs straight out from Harry's starting point. Harry runs along the ray $\varphi = 0$ for the time $t_1 = d/(V - v)$, reaching the distance $\alpha d/(\alpha - 1)$. He then keeps his distance from the start equal to $d + vt$ and turns through 2π. He meets every ship, and the whole chase takes

$$T_{\text{spiral}} = \frac{\alpha d}{v(\alpha - 1)}\, e^{2\pi/\sqrt{\alpha^2 - 1}} - \frac{d}{v}.$$

Status: CITED (classical; not proved in this book). Our check, COMPUTED: the formula equals a direct integration of the turn to 15 digits for $\alpha \in \{1.25, 1.5, 2, 3, 5, 7\}$. For $d = v = 1$ the times are 21741.37, 826.55, 74.24, 12.83, 3.51 and 1.89.

Proof: chapters/02-fog-and-spirals.md; results file certificates/results.json, entry "spiral_time".

Theorem 2: the exact path of a Snitch while Harry runs straight (Bouguer 1732, run backwards in time; Sonny, 2026)

Credit: Builds on: the classical pursuit-curve integral of Bouguer (1732), run backwards in time. We checked that mapping and did not check it against a printed formula. This result (Sonny): proved (written).

Let Harry move on a straight line, $H(t) = H_0 + V(t - t_0)\hat{u}$, and let the Snitch be uncaught. Write $S - H = r(\cos\theta, \sin\theta)$, with $\theta \in (0, \pi)$ measured from $\hat{u}$ (the case $\theta \in (-\pi, 0)$ is the mirror image), and $u = \tan(\theta/2)$. Then $\dot{r} = v - V\cos\theta$ and $r\dot{\theta} = V\sin\theta$, and

$$r\sin\theta = y_0\, u_0^{-1/\alpha}\, u^{1/\alpha}, \qquad r(u) = \frac{y_0}{2u_0^{1/\alpha}}\left(u^{1/\alpha - 1} + u^{1/\alpha + 1}\right),$$

$$V(t - t_0) = \frac{y_0}{2u_0^{1/\alpha}}\left[\frac{\alpha}{\alpha - 1}\left(u_0^{1/\alpha - 1} - u^{1/\alpha - 1}\right) + \frac{\alpha}{\alpha + 1}\left(u^{1/\alpha + 1} - u_0^{1/\alpha + 1}\right)\right],$$

where $y_0 = r_0\sin\theta_0$ and $u_0 = \tan(\theta_0/2)$.

Status: PROVED (written proof, independently re-derived). The proof divides the two equations, integrates $d\theta/\sin\theta = du/u$, and substitutes $\sin\theta = 2u/(1 + u^2)$. The formulas were re-derived independently. COMPUTED: against direct numerical integration, the largest difference over 25 cases ($\alpha \in \{1.25, 1.5, 2, 3, 5\}$, $\varphi \in \{0.1, 0.5, 1, 2, 2.8\}$, at time $t_1 = d/(V - v)$) is 9.9×10^{-14}.

Proof: chapters/03-what-we-proved.md, section "One Snitch, exactly"; recipe 2 in VERIFY.md.

Theorem 3: the corner at the end of the opening run

Credit: This result (Sonny): proved (written).

Harry runs from the origin along the positive x-axis. Let $t_1 = d/(V - v)$ and let S_φ be the Snitch that starts at angle $\varphi \in (0, \pi)$. As $\varphi \to 0^+$, at time t_1,

$$\tan\frac{\theta}{2} \to u_* = \sqrt{\frac{\alpha+1}{\alpha-1}}, \qquad \cos\theta \to -\frac{1}{\alpha},$$

$$r_1(\varphi) = \| S_\varphi(t_1) - H(t_1) \| = C(\alpha)\, d\, \varphi^{1-1/\alpha}\,(1 + o(1)),$$

$$\sigma(\varphi, t_1) = \| \partial_\varphi S_\varphi(t_1) \| = \left(1 - \frac{1}{\alpha}\right) C(\alpha)\, d\, \varphi^{-1/\alpha}\,(1 + o(1)),$$

with $C(\alpha) = 2^{1/\alpha - 1}\left(u_*^{1/\alpha - 1} + u_*^{1/\alpha + 1}\right)$. So the curve of possible positions leaves $H(t_1)$ along two arms, each at the angle $\arccos(1/\alpha)$ from the direction straight behind Harry. The exponent $-1/\alpha$ is above -1, so σ is integrable near $\varphi = 0$.

Status: PROVED (written proof, independently re-derived). The proof sets $t = t_1$ in Theorem 2, uses $Vt_1 = \alpha d/(\alpha - 1)$, $y_0 = d\sin\varphi$ and $u_0 = \tan(\varphi/2)$, and lets $\varphi \to 0^+$; the leading terms cancel and leave $u_*^2 = (\alpha + 1)/(\alpha - 1)$. The algebra was re-derived by hand independently. COMPUTED: at $\varphi = 10^{-6}$ the error in $\cos\theta + 1/\alpha$ is at most 2.6×10^{-8}, and the ratios of r_1 and σ to their formulas are within 1.8×10^{-7} of 1, for $\alpha \in \{1.25, 1.5, 2, 3, 5\}$.

Proof: chapters/03-what-we-proved.md, section "The corner"; recipe 3 in VERIFY.md.

Result 4: the length of the curve at the end of the opening run

Credit: This result (Sonny): computed.

At time $t_1 = d/(V - v)$ the curve of possible positions is shorter than the circle $\| x \| = d + vt_1$. In units of d, the curve has length 26.8575 against 31.4159 at $\alpha = 1.25$; 11.7249 against 12.5664 at $\alpha = 2$; and 7.6817 against 7.8540 at $\alpha = 5$.

Status: COMPUTED, at these three speed ratios only. A proof for every ratio is not in this book.

Proof: results file certificates/results.json, entry "length_at_t1"; recipe 4 in VERIFY.md.

Theorem 5: the tip law

Credit: This result (Sonny): proved (written).

Conditions (T). For $t \geq t_1$ the uncaught Snitches form a regular arc $S(\varphi, t)$, $\varphi \in [\Phi(t), 2\pi]$, twice continuously differentiable in φ up to the tip $H(t) = S(\Phi(t), t)$, with $\sigma = \| \partial_\varphi S \| > 0$ and finite curvature κ at the tip; $\partial_t S$ has a one-sided limit at the tip; and Harry moves along the unit tangent $\hat{\tau}$ at the tip at speed V. Let $\rho = \| S - H \|$, $\hat{r} = (S - H)/\rho$, and let β be the angle between $\hat{\tau}$ and $\hat{r}$. Under (T):

1. $\dot{\Phi}(t) = \dfrac{V - v}{\sigma(\Phi(t), t)}$, so the arc length consumed at the tip per unit time is $V - v$.
2. At every point of the arc, $\dfrac{1}{\sigma}\dfrac{\partial \sigma}{\partial t} = \dfrac{v \sin^2 \beta}{\rho}$.
3. Near the tip, with s the arc length from the tip, this rate equals $\frac{1}{4} v \kappa^2 s + O(s^2)$, which tends to 0 as $s \to 0^+$.

Status: PROVED (written proof, independently re-derived) under (T). The conditions (T) are not shown for the real chase, whose uncaught curve has the corner of Theorem 3 at t_1. COMPUTED: part 2 matched a finite-difference test on 5 random arcs, with relative error at most 3.7×10^{-5}.

Proof: chapters/04-the-chase-along-the-edge.md, section "Surfing the edge"; results file certificates/results.json, entry "stretching_identity".

Theorem 6: the length balance, and capture for $\alpha > \pi + 1$ if a condition holds

Credit: This result (Sonny): proved (written).

Assume (T) of Theorem 5 on $[t_1, T)$. Let $L(t) = \int_{\Phi(t)}^{2\pi} \sigma(\varphi, t)\, d\varphi$ be the length of the uncaught arc, and let $\psi(\varphi, t)$ be the polar angle of $S - H$. Then:

1. $\dfrac{dL}{dt} = -(V - v) + v \displaystyle\int_{\Phi(t)}^{2\pi} \sin\beta\, \partial_\varphi \psi \, d\varphi$, and the integrand is nonnegative, so $dL/dt \geq -(V - v)$. A chase of this kind therefore needs at least $t_1 + L(t_1)/(V - v)$.
2. If, in addition, for every $t \in [t_1, T)$ the map $\varphi \mapsto \psi(\varphi, t)$ is monotone on $[\Phi(t), 2\pi]$ with total change at most π (condition (A)), then $dL/dt \leq -(\alpha - \pi - 1)\, v$. For $\alpha > \pi + 1$, L reaches 0 no later than $t_1 + L(t_1)/\Big((\alpha - \pi - 1)v\Big)$, and every Snitch has been caught.

Status: PROVED (written proof, independently re-derived) as an implication. Neither (T) nor (A) is shown for the real chase, so capture for $\alpha > \pi + 1$ is conditional on them. In every finished sample chase of Result 8 the total change of ψ stayed below π (largest value 3.083), and in the runs at $\alpha = 1.25$ and 1.5 it exceeded π before the positions ran away.

Proof: chapters/04-the-chase-along-the-edge.md, section "Counting the length left". The step uses $\sigma \sin\beta = \rho\, \partial_\varphi \psi$, checked in a finite-difference test with relative error at most 2.4×10^{-5} (COMPUTED).

Remark 7: the sawtooth step, a heuristic

Credit: This result (Sonny): proved (written).

Take the linearised edge equation $\partial_t \eta = (V - v)\, \partial_s \eta + (v/s)\, \eta - V\, \partial_s \eta(0, t)$ for a small transverse wobble η at distance s from the tip. In a simple forward step of length $\Delta t = \Delta s/(V - v)$ on a grid of spacing Δs, the sample next to the tip, η_1, takes the updated value η_1^+ given by

$$\eta_1^+ = \left(1 + \frac{1}{2(\alpha - 1)}\right)\eta_2 - \frac{\alpha}{\alpha - 1}\,\eta_1.$$

The tip's own coefficient is $-\alpha/(\alpha - 1)$. A wobble that alternates in sign, $\eta_j = (-1)^j$, is multiplied by $-(4\alpha - 1)/(2(\alpha - 1))$ in one step, which is -8 at $\alpha = 1.25$ and -3.5 at $\alpha = 2$.

Status: PROVED for the one-step arithmetic, given the equation; the equation is our linearisation and was not independently derived. Reading this step as the reason the sample chases fail at $\alpha \leq 1.5$ is a heuristic only: both coefficients are larger than 1 in size for every $\alpha > 1$, so the step does not by itself separate $\alpha = 1.5$ from $\alpha = 2$.

Proof: chapters/04-the-chase-along-the-edge.md, section "A warning from the numbers".

Result 8: chases of sample Snitches with the exact motion

Credit: This result (Sonny): computed.

Harry runs the opening run of length $t_1 = d/(V - v)$. He then catches N sample Snitches one after another, always flying straight at the next, with every remaining sample moving by the exact formula of Theorem 2 and no smoothing. The samples start at the angles $2\pi k/N$, $k = 0, \ldots, N - 1$. For $d = v = 1$ the chase ends at these times:

α	$N = 180$	$N = 360$	$N = 720$	$N = 1440$
2	22.5334	22.9100	23.2023	23.4303
3	7.0694	7.1337	7.1770	7.2067
5	2.6143	2.6281	2.6362	2.6410
7	1.5549	1.5614	1.5650	1.5671

At $\alpha = 1.25$ and $\alpha = 1.5$ the chase does not finish: the positions run away, for $N \in \{180, 360, 720\}$ on this layout and for $M \in \{200, 400, 800\}$ on a layout that grades the starting angles toward the corner.

Status: COMPUTED, for these sample sets. The times rise each time N doubles, so no limit is shown. Catching N sample Snitches does not imply catching all of them. Schemes that smooth the Snitch positions do finish at $\alpha = 1.5$, but smoothing changes the positions, so their times are not capture times for any real path, and none is reported here.

Proof: results file certificates/results.json, entries "chase_uniform_start" and "chase_graded_start"; recipe 5 in VERIFY.md.

Conjecture 9: point capture for $1 < \alpha \leq \pi + 1$

Credit: Builds on: the question of point capture for $\alpha > 1$, as posed by Don't Panic Math.

For every α with $1 < \alpha \leq \pi + 1$ there is a path of Harry that catches the Snitch exactly, whatever its starting angle φ.

Status: CONJECTURED. The numerical support starts at $\alpha = 2$: the sample chases of Result 8 finish at $\alpha = 2$ and 3. There is none below $\alpha = 2$. For $\alpha > \pi + 1$ the question is conditional (Theorem 6), and for all $\alpha > 1$ it is open.

Proof: none. chapters/05-the-open-question.md.

10 Verify: The Invisible Golden Snitch

Verify: The Invisible Golden Snitch

This guide gives a written recipe for each check, so that you can check the results with a few lines of code of your own. Each recipe states the numbers to start from, the formula, the expected answer, and what a pass shows and does not show. No program of ours is needed; the numbers below are enough to reproduce each result with a few lines of your own code. Each recipe also comes as a step-by-step page with commands for Windows, macOS and Linux and the exact output to expect.

What is claimed

Question	Claim	Range
How long does the classical spiral take?	$T = \frac{\alpha d}{v(\alpha - 1)} e^{2\pi/\sqrt{\alpha^2 - 1}} - d/v$ equals a direct integration of the turn	6 speed ratios
Is the closed form of a Snitch's path right?	It matches direct numerical integration of the motion	25 start angles and speed ratios
Is the corner as stated?	At a start angle of 10^{-6}, $\cos\theta = -1/\alpha$ and the two power laws hold to better than 2×10^{-7}	5 speed ratios
Is the curve shorter than the circle?	Yes at the three ratios measured	3 speed ratios
Does the chase of sample Snitches finish?	Yes at $\alpha = 2, 3, 5, 7$. No at $\alpha = 1.25, 1.5$	4 sample sizes, two layouts
Can Harry catch the Snitch exactly, at every speed ratio above 1?	Not shown here (status OPEN)	the whole question

The statements, with their conditions, are in the statement file and in THEOREMS.md.

Status

Computed within this project; not yet refereed. An independent replay of the formulas and of the chases agreed with the numbers below. Nobody outside this project has reviewed any of this.

- Closed form and corner: proved on paper (Theorems 2 and 3) and checked numerically (recipes 2 and 3).
- Tip law and length balance: proved under stated conditions (Theorems 5 and 6). The conditions are not shown for the real chase. Recipe 6 checks only the stretching identity.
- Chases: computed (recipe 5). They are samples, not the whole curve.

Commands

Step 0: fingerprints

Run inside the certificates folder. Every line must report OK. Any program that computes SHA-256 will do; on Windows use the second command for one file at a time.

```
sha256sum -c SHA256SUMS.txt  # Linux
shasum -a 256 -c SHA256SUMS.txt   # macOS
CertUtil -hashfile results.json SHA256
```

The folder holds one results file with every number quoted in the book. The recipes below let you reproduce each number without it.

Recipe 1: the spiral time

Take $d = v = 1$ and $V = \alpha$. Harry's turn starts at the distance $\alpha/(\alpha - 1)$ at time $1/(\alpha - 1)$. Along the turn his distance from the start is $1 + t$ and his angle grows at the rate $\sqrt{\alpha^2 - 1}\,/(1 + t)$. Find the time T at which the angle reaches 2π, by integrating or by solving $\sqrt{\alpha^2 - 1}\,\ln\left((1 + T)/(1 + 1/(\alpha - 1))\right) = 2\pi$. Compare with the formula.

Expected: $T = 21741.3733,\ 826.5455,\ 74.2447,\ 12.8309,\ 3.5073,\ 1.8894$ for $\alpha = 1.25, 1.5, 2, 3, 5, 7$. The two ways agree to 15 digits. A pass shows the formula is the right integral of the turn. It does not show that the spiral is an answer to the riddle, whose Snitch flees from Harry's current position.

Recipe 2: the closed form against direct integration

Take $d = v = 1$, $V = \alpha = 2$, and a Snitch starting at $(\cos 1, \sin 1)$. Harry is at $(Vt, 0)$. Integrate $\dot{S} = (S - H)/\parallel S - H \parallel$ numerically up to $t_1 = 1/(\alpha - 1) = 1$ with a tight tolerance (relative 10^{-13}). Separately, use Theorem 2: put $\theta_0 = 1$, $r_0 = 1$, $y_0 = \sin 1$, $u_0 = \tan(1/2)$, solve the time formula for u so that $V(t_1 - 0)$ is matched (the time grows with u, so bisection works), compute r and θ, and place the Snitch at $H(t_1) + r(\cos\theta, \sin\theta)$.

Expected: both give $(0.33617353, 1.74108756)$, with $r = 2.4082576$ and $\theta = 2.3335073$. Over the 25 cases of the claims table the largest difference is 9.9×10^{-14}. A pass shows the formula solves the motion. It does not show anything about the chase.

Recipe 3: the corner

With $d = v = 1$ and $t_1 = 1/(\alpha - 1)$, use the recipe 2 formulas at the start angle $\varphi = 10^{-6}$. Compute $\cos\theta$, the distance r_1 from Harry, and σ as the distance between the positions for $\varphi(1 \pm 10^{-4})$ divided by $2\varphi \cdot 10^{-4}$. Compare with $-1/\alpha$, with $C\varphi^{1-1/\alpha}$ and with $(1 - 1/\alpha)C\varphi^{-1/\alpha}$, where $u_* = \sqrt{(\alpha+1)/(\alpha-1)}$ and $C = 2^{1/\alpha - 1}(u_*^{1/\alpha - 1} + u_*^{1/\alpha + 1})$.

Expected, at $\alpha = 2$: $C = 2.14913986$, $\cos\theta = -0.50000000023$, and both ratios equal 1 to within 10^{-8}. The arms leave at $\arccos(1/\alpha)$ from straight behind: 36.87, 48.19, 60.00, 70.53 and 78.46 degrees for $\alpha = 1.25, 1.5, 2, 3, 5$. A pass shows the limits are right to the precision tested. It does not show them for other ratios.

Recipe 4: the length of the curve

At $\alpha = 2$, $d = v = 1$, $t_1 = 1$: compute the positions at t_1 for $\varphi_i = \pi(i/2000)^4$, $i = 1, \ldots, 1999$, and for $\varphi = \pi - 10^{-9}$, by recipe 2. Put the tip $(\alpha t_1, 0)$ in front. Add up the straight distances between neighbouring points, and double the sum (the curve is symmetric about the x-axis).

Expected: 11.7249, against the circle's $2\pi(1 + t_1) = 12.5664$. At $\alpha = 1.25$: 26.8575 against 31.4159. At $\alpha = 5$: 7.6817 against 7.8540. A pass shows the curve is shorter than the circle at these three ratios. It does not show it for other ratios.

Recipe 5: the chase of sample Snitches

Take $d = v = 1$, $V = \alpha$, and N samples at the angles $\varphi_k = 2\pi k/N$, $k = 0, \ldots, N-1$.

1. Harry flies along $+x$ for $t_1 = 1/(\alpha - 1)$. Move samples $1, \ldots, N-1$ by "the move" below, which is recipe 2 applied to a sample. Harry is at $(\alpha t_1, 0)$ and sample 0 is caught.
2. For $k = 1, \ldots, N-1$: let $R = S_k - H$, $\ell = \| R \|$, $\hat{u} = R/\ell$, $\Delta t = \ell/(V - v)$. Move samples $k+1, \ldots, N-1$ for the time Δt while Harry flies along $\hat{u}$. Set $H \leftarrow H + V\Delta t\,\hat{u}$, $S_k \leftarrow H$ and $t \leftarrow t + \Delta t$.
3. The answer is the final t, which started at t_1. If any distance exceeds 10^{15}, the chase does not finish.

The move, for one sample while Harry flies straight along $\hat{u}$ for the time Δt: write the sample as (x, y) in the frame whose x-axis is $\hat{u}$, with Harry at the origin. If $y \neq 0$, use Theorem 2 with $\theta_0 = \operatorname{atan2}(|y|, x)$ and $y_0 = |y|$, find u at the later time, and keep the sign of y. If $y = 0$ and $x \geq 0$ the sample is dead ahead and the gap closes at $V - v$; if $x < 0$ it recedes at $V + v$.

Expected for $N = 180$: 22.5334, 7.0694, 2.6143, 1.5549 at $\alpha = 2, 3, 5, 7$, and no finish at $\alpha = 1.25, 1.5$. For $N = 1440$: 23.4303, 7.2067, 2.6410, 1.5671. The largest total turn of the directions from Harry to the remaining samples is 2.92, 3.02, 3.07 and 3.08 at $N = 1440$. A pass shows these times for these sample sets. It does not show a limit as N grows, and it does not show that every Snitch is caught.

Recipe 6: the stretching identity

Take a smooth arc $S(\varphi)$ and a point H off it, and let each point flee from H at speed v for a short time h (use $h = 10^{-6}$, with a second-order step for the flow). Compute $\sigma = \| \partial_\varphi S \|$ by finite differences before and after, in both time directions, and take $(\sigma_{+h} - \sigma_{-h})/(2h)$. Compare $\sigma^{-1}\partial_t \sigma$ with $v \sin^2 \beta / \rho$, where $\rho = \| S - H \|$ and β is the angle between $\partial_\varphi S$ and $S - H$.

Expected: relative error at most 3.7×10^{-5} on five random arcs, which is the level of the finite differences. A pass shows the identity of Theorem 5, part 2. It does not show that the conditions (T) hold for the real chase.

Expected output

Step 0 prints the line below. Recipes 1 to 6 give the numbers above. The results file lists the same numbers under these entry names.

```
results.json: OK

spiral_time
closed_form_vs_integration
corner
length_at_t1
stretching_identity
sawtooth_algebra
chase_uniform_start
chase_graded_start
```

Time and memory

Recipes 1 to 4 and 6 take seconds on one core. Recipe 5 at $N = 1440$ takes under a minute per speed ratio, and memory use is a few megabytes.

What the checks do not show

- A catch of the whole curve: sample chases leave Snitches between samples.
- Conditions (T) and (A) for the real chase.
- Any capture result at $\alpha \leq 1.5$.
- The video's claim of disk capture for $\alpha > 2\pi$.

What is not shipped, and why

The programs that produced the results file are not shipped. The recipes above are enough to reproduce every number.

Our own programs and methods

The programs are not part of this book.

Where to go next

Which number would you check? Run recipe 2, then see the theorem list for what it proves, and the open question for what is left.

We hope it helps others find more results and keeps their curiosity going.
We'd love to collaborate with anyone.

11 Files and fingerprints

This chapter is the book's file SHA256SUMS.txt.

12 Trust and verification

This chapter is the book's file VERIFY.md.

13 People and works

This chapter is the book's file CREDITS.md.

14 Files and rights

Licence

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What it covers: the chapters, the statements, the figures, and the results file with the numbers the book quotes.

What it does not cover:

- The puzzle, which belongs to Don't Panic Math, who posed it in the video "The Invisible Golden Snitch Riddle". We describe it with credit, and we do not copy the video or its slides.

- NumPy, SciPy and Matplotlib, under their own licences. They are not shipped; a reader installs them under those licences. The programs that produced and checked the results are not part of this book.
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Where to go next

- What does this book show? See the book's front page.

15 Each result on its own page

1. Proposition 1: the spiral against ships that run straight out STATUS: CITED
2. Theorem 2: the exact path of a Snitch while Harry runs straight STATUS: PROVED IN WRITING
3. Theorem 3: the corner at the end of the opening run STATUS: PROVED IN WRITING
4. Result 4: the length of the curve at the end of the opening run STATUS: COMPUTED
5. Theorem 5: the tip law STATUS: PROVED IN WRITING
6. Theorem 6: the length balance, and capture for $\alpha > \pi + 1$ if a condition holds STATUS: PROVED IN WRITING
7. Remark 7: the sawtooth step, a heuristic STATUS: PROVED IN WRITING; THE PROGRAM IS NOT PUBLISHED HERE
8. Result 8: chases of sample Snitches with the exact motion STATUS: COMPUTED; THE PROGRAM IS NOT PUBLISHED HERE

9. Conjecture 9: point capture for $1 < \alpha \leq \pi + 1$ STATUS: OPEN (CONJECTURED)

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